3.18)

S cumple: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

→ $\bar{X}$ que cumplen tienen la forma:

$$\bar{X} = \begin{bmatrix} a & b \\ b & d \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + d \cdot \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

→ $B_S = \left\{ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{v_1}, \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{v_2}, \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{v_3} \right\}$ ya q' son LI.

~~Esta base cumple que: $v_1 \cdot v_2 = 0$, $v_1 \cdot v_3 = 0$, $v_2 \cdot v_3 = 0$ con el PIC de $\mathbb{R}^{2\times2}$, por lo que es ortogonal.~~

Verifico si es ortogonal:

Para q' sea ortogonal: $\underbrace{\langle v_1, v_2 \rangle = 0}_{\mathrm{I}}$, $\underbrace{\langle v_1, v_3 \rangle = 0}_{\mathrm{II}}$, $\underbrace{\langle v_2, v_3 \rangle = 0}_{\mathrm{III}}$.

I $\langle v_1, v_2 \rangle = tr(v_2^* v_1) = tr\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = tr\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = 0$ ✓

II $\langle v_1, v_3 \rangle = tr(v_3^* v_1) = tr\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = tr\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right) = 0$ ✓

III $\langle v_2, v_3 \rangle = tr(v_3^* v_2) = tr\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) = tr\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = 0$ ✓

B_S Es ortogonal ✓

Puedo aplicar la fórmula de proyección.

a) $P_S\left(\underbrace{\begin{bmatrix}a&b\\c&d\end{bmatrix}}_{X}\right) = \dfrac{\left\langle \begin{bmatrix}a&b\\c&d\end{bmatrix}, \overbrace{\begin{bmatrix}1&0\\0&0\end{bmatrix}}^{v_1}\right\rangle}{\left\|\begin{bmatrix}1&0\\0&0\end{bmatrix}\right\|^2}\cdot\begin{bmatrix}1&0\\0&0\end{bmatrix} + \dfrac{\left\langle \begin{bmatrix}a&b\\c&d\end{bmatrix}, \overbrace{\begin{bmatrix}0&1\\1&0\end{bmatrix}}^{v_2}\right\rangle}{\left\|\begin{bmatrix}0&1\\1&0\end{bmatrix}\right\|^2}\cdot\begin{bmatrix}0&1\\1&0\end{bmatrix} + \dfrac{\left\langle \begin{bmatrix}a&b\\c&d\end{bmatrix}, \overbrace{\begin{bmatrix}0&0\\0&1\end{bmatrix}}^{v_3}\right\rangle}{\left\|\begin{bmatrix}0&0\\0&1\end{bmatrix}\right\|^2}\cdot\begin{bmatrix}0&0\\0&1\end{bmatrix}$

$\rightarrow P_S(X) = \begin{bmatrix}a&0\\0&0\end{bmatrix} + \begin{bmatrix}0&\frac{c+b}{2}\\\frac{c+b}{2}&0\end{bmatrix} + \begin{bmatrix}0&0\\0&d\end{bmatrix} = \begin{bmatrix}a&\frac{c+b}{2}\\\frac{c+b}{2}&d\end{bmatrix} \Rightarrow$

$\rightarrow \boxed{P_S(X) = \begin{bmatrix}a&\frac{c+b}{2}\\\frac{c+b}{2}&d\end{bmatrix}}$

b) $\boxed{P_S\left(\underbrace{\begin{bmatrix}1&-1\\1&1\end{bmatrix}}_{B}\right) = \begin{bmatrix}1&\frac{1-1}{2}\\\frac{1-1}{2}&1\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix}}$

$d(B,S) = \|B - P_S(B)\| = \underbrace{\left\|\begin{bmatrix}0&-1\\1&0\end{bmatrix}\right\|}_{\text{I}}$

I $\rightarrow \left\|\begin{bmatrix}0&-1\\1&0\end{bmatrix}\right\|^2 = \left\langle\begin{bmatrix}0&-1\\1&0\end{bmatrix}, \begin{bmatrix}0&-1\\1&0\end{bmatrix}\right\rangle = tr\left(\begin{bmatrix}0&1\\-1&0\end{bmatrix}\cdot\begin{bmatrix}0&-1\\1&0\end{bmatrix}\right) = tr\left(\begin{bmatrix}1&0\\0&1\end{bmatrix}\right) = 2$

$\rightarrow \boxed{\left\|\begin{bmatrix}0&-1\\1&0\end{bmatrix}\right\| = \sqrt{2}}$

dist. de B a S.